

A Rigorous Method for Computation of Ferrite Toroidal Phase Shifters

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Abstract—In this paper, coupled wave theory [1] is used to compute ferrite toroidal phase shifters. Computation results show that this method is very effective, rather simple, and easy to handle. As an example, a computation is carried out to analyze the twin toroidal model, which can be readily produced with considerably larger phase shift than the commonly used single toroidal model. Experimental results are in good agreement with theoretical analysis. Our research work shows that coupled wave theory is a powerful method for treating electromagnetic problems of waveguides loaded with magnetized ferrites.

I. INTRODUCTION

ALTHOUGH FERRITE toroidal phase shifters have found widespread applications in phase array antennas and have been studied and developed intensively [2]–[8], rigorous computation is lacking since it is necessary to solve complicated boundary value problems of waveguides loaded with magnetized ferrites. The commonly used twin slab model is simple and effective [8], but it is approximate and must be corrected to allow for the effect of the upper and lower parts of the toroid which are in contact with the broad walls of the waveguide. Although some correcting methods have been introduced [5], discrepancy between calculation and experiment remains. In this paper, coupled wave theory is used to calculate the performance of twin toroidal phase shifters; this was first suggested by Ince and Stern [4]. Theoretical and experimental results show that this method is very effective and that twin toroidal phase shifters have many advantages over the single toroidal model. Calculated results are in good agreement with experiments.

II. THEORETICAL ANALYSIS

The construction of a twin toroidal phase shifter is shown in Fig. 1. In this case the rectangular waveguide is filled with transversely magnetized ferrites and the configuration of the electromagnetic fields is very complicated since all six field components are present. From the coupled wave theory [1], the fields in this waveguide can be represented by the summation of eigenmodes of the empty

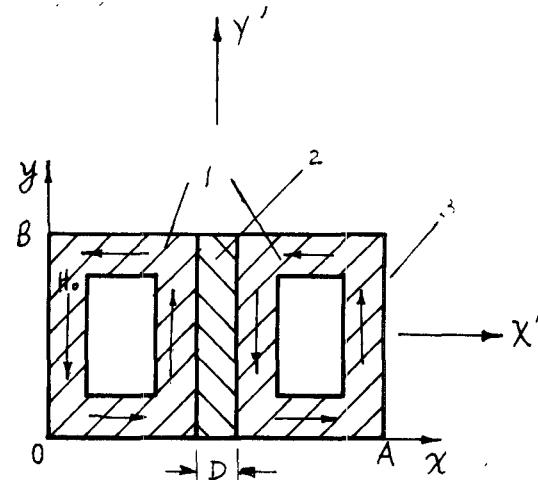


Fig. 1. Construction of twin phase shifter (1—ferrite toroid, 2—dielectric slab, 3—waveguide).

waveguide, i.e.,

$$\begin{aligned}
 E_x &= \sum_{K=1}^{\infty} \left(V_K \frac{\partial \Pi_K}{\partial x} + V_K^* \frac{\partial \Pi_K^*}{\partial y} \right) \\
 E_y &= \sum_{K=1}^{\infty} \left(V_K \frac{\partial \Pi_K}{\partial x} - V_K^* \frac{\partial \Pi_K^*}{\partial y} \right) \\
 E_z &= \sum_{K=1}^{\infty} V_{z,K} \Pi_K \\
 H_x &= \sum_{K=1}^{\infty} \left(-I_K \frac{\partial \Pi_K}{\partial y} + I_K^* \frac{\partial \Pi_K^*}{\partial x} \right) \\
 H_y &= \sum_{K=1}^{\infty} \left(I_K \frac{\partial \Pi_K}{\partial x} + I_K^* \frac{\partial \Pi_K^*}{\partial y} \right) \\
 H_z &= \sum_{K=1}^{\infty} I_{z,K} \Pi_K^* \tag{1}
 \end{aligned}$$

where Π_K, Π_K^* are Hertzian functions of the electric waves (TM modes) and magnetic waves (TE modes) of the empty rectangular waveguide, respectively. They may be expressed as follows.

$$\begin{aligned}
 \Pi_K &= \Lambda_K \sin \frac{p\pi}{a} x \sin \frac{q\pi}{b} y \\
 \Pi_K^* &= \Lambda_K^* \cos \frac{m\pi}{a} x \cos \frac{n\pi}{b} y
 \end{aligned}$$

where Λ_K, Λ_K^* are normalizing coefficients that satisfy the

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equations

$$\chi_K^2 \int_S \Pi_K^2 dS = \chi_K^{*2} \int_S \Pi_K^{*2} dS = 1$$

where S is the cross section of the waveguide and $V_K, V_K^*, V_{z,K}, I_K, I_K^*, I_{z,K}^*$ are coefficients of expansion, i.e., voltage and current coefficients of electric and magnetic waves, respectively.

The tensor permeability of the ferrite, magnetized to remanence in the $x-y$ plane, is given by

$$\hat{\mu}(x, y) = \begin{bmatrix} \mu & 0 & jk\gamma_y \\ 0 & \mu & jk\gamma_x \\ -jk\gamma_y & -jk\gamma_x & \mu \end{bmatrix} \quad (2)$$

where γ_x and γ_y are the projections of the unit vector along the direction of magnetization on the coordinates x and y , respectively; γ_x and γ_y are functions of the coordinates of x and y and we have

$$\gamma_x^2 + \gamma_y^2 = 1 \quad \text{and} \quad \mu \approx 1.$$

In our calculation the ferrite is assumed to be lossless.

Since the waveguide is loaded inhomogeneously with magnetized ferrite in the $x-y$ plane, Maxwell's equations take the following form:

$$\begin{aligned} \nabla \times \vec{H} &= j\omega\epsilon_0\epsilon(x, y)\vec{E} \\ \nabla \times \vec{E} &= -j\omega\mu_0\hat{\mu}(x, y)\vec{H}. \end{aligned} \quad (3)$$

Substituting (1) and (2) into (3) and using the orthogonality of Hertzian functions we obtain

$$\begin{aligned} -j\frac{dV_i}{dz} &= \sum_{K=1}^{\infty} Z_{[i][K]} I_K + \sum_{K=1}^{\infty} Z_{[i](K)} I_K^* \\ &\quad + \sum_{K=1}^{\infty} T_{[i][K]}^V V_K + \sum_{K=1}^{\infty} T_{[i](K)}^V V_K^* \\ -j\frac{dV_i^*}{dz} &= \sum_{K=1}^{\infty} Z_{(i)[K]} I_K + \sum_{K=1}^{\infty} Z_{(i)(K)} I_K^* \\ &\quad + \sum_{K=1}^{\infty} T_{(i)[K]}^V V_K + \sum_{K=1}^{\infty} T_{(i)(K)}^V V_K^* \\ -j\frac{dI_i}{dz} &= \sum_{K=1}^{\infty} T_{[i][K]}^I I_K + \sum_{K=1}^{\infty} T_{[i](K)}^I I_K^* \\ &\quad + \sum_{K=1}^{\infty} Y_{[i][K]} V_K + \sum_{K=1}^{\infty} Y_{[i](K)} V_K^* \\ -j\frac{dI_i^*}{dz} &= \sum_{K=1}^{\infty} T_{(i)[K]}^I I_K + \sum_{K=1}^{\infty} T_{(i)(K)}^I I_K^* \\ &\quad + \sum_{K=1}^{\infty} Y_{(i)[K]} V_K + \sum_{K=1}^{\infty} Y_{(i)(K)} V_K^* \end{aligned} \quad (4)$$

where Z , Y , T^I , and T^V are transfer impedances, transfer admittances, and the voltage and current transfer coeffi-

cients, respectively [1]. They can be expressed as follows:

$$\begin{aligned} Z_{[i][K]} &= (\chi_i^2 \chi_K^2 / \omega \epsilon_0) \int_S [\Pi_i \Pi_K / \epsilon(x, y)] dS - \omega \mu_0 \delta_{iK} \\ &\quad + \omega \mu_0 k^2 \int_S [(\vec{G}(x, y) \times \vec{a}_z) \cdot \nabla_i \Pi_K] \\ &\quad \cdot [(\vec{G}(x, y) \times \vec{a}_z) \cdot \nabla_i \Pi_i] dS \\ Z_{[i](K)} &= \omega \mu_0 k^2 \int_S (\vec{G}(x, y) \cdot \nabla_i \Pi_K^*) \\ &\quad \cdot [(\vec{G}(x, y) \times \vec{a}_z) \cdot \nabla_i \Pi_K] dS \\ Z_{(i)[K]} &= \omega \mu_0 k^2 \int_S (\vec{G}(x, y) \cdot \nabla_i \Pi_i^*) \\ &\quad \cdot [(\vec{G}(x, y) \times \vec{a}_z) \cdot \nabla_i \Pi_K] dS \\ Z_{(i)(K)} &= \omega \mu_0 k^2 \int_S (\vec{G}(x, y) \cdot \nabla_i \Pi_i^*) \\ &\quad \cdot (\vec{G}(x, y) \cdot \nabla_i \Pi_K^*) dS - \omega \mu_0 \delta_{iK} \\ Y_{[i][K]} &= -\omega \epsilon_0 \int_S \epsilon(x, y) (\nabla_i \Pi_i \cdot \nabla_i \Pi_K) dS \\ Y_{[i](K)} &= -\omega \epsilon_0 \int_S \epsilon(x, y) \nabla_i \Pi_i \cdot (\nabla_i \Pi_K^* \times \vec{a}_z) dS \\ Y_{(i)[K]} &= \omega \epsilon_0 \int_S \epsilon(x, y) \nabla_i \Pi_i^* \cdot [\nabla_i \Pi_K \times \vec{a}_z] dS \\ T_{[i][K]}^V &= k \chi_K^{*2} \int_S \Pi_K^* [\nabla_i \Pi_i \cdot (\vec{G}(x, y) \times \vec{a}_z)] dS \\ T_{(i)[K]}^V &= k \chi_K^{*2} \int_S \Pi_K^* (\vec{G}(x, y) \cdot \nabla_i \Pi_i^*) dS \\ T_{[i][K]}^I &= T_{(i)[K]}^V = 0 \\ T_{(i)[K]}^I &= T_{[K](i)}^V \\ T_{[i](K)}^I &= T_{[i][K]}^V = 0. \end{aligned}$$

Here S is the cross section of the waveguide (see Fig. 1) and

$$\vec{G}(x, y) = \gamma_y \vec{a}_x + \gamma_x \vec{a}_y \quad \delta_{iK} = \begin{cases} 1 & \text{for } i = K \\ 0 & \text{for } i \neq K \end{cases}$$

$$\chi_i^2 = \chi_i^{*2} = \pi^2 (m^2/a^2 + n^2/b^2)$$

\vec{a}_x , \vec{a}_y , and \vec{a}_z are the unit vectors along the x , y , and z axes, respectively.

Putting $\partial/\partial z = -j\beta$ and selecting a finite number of electric and magnetic waves (i.e., to truncate the series in (1)), this system of equations may be transformed into

matrix form:

$$-\beta \begin{pmatrix} I \\ I^* \\ V \\ V^* \end{pmatrix} = \begin{pmatrix} 0_{L \times L} & 0_{L \times M} & \tilde{Y}_{[L][L]} & \tilde{Y}_{[L][M]} \\ \tilde{T}'_{(M)[L]} & \tilde{T}'_{(M)(M)} & \tilde{Y}_{(M)[L]} & \tilde{Y}_{(M)(M)} \\ \tilde{Z}_{[L][L]} & \tilde{Z}_{[L][M]} & 0_{L \times L} & \tilde{T}'_{[L][M]} \\ \tilde{Z}_{(M)[L]} & \tilde{Z}_{(M)(M)} & 0_{M \times L} & \tilde{T}'_{(M)(M)} \end{pmatrix} \cdot \begin{pmatrix} I \\ I^* \\ V \\ V^* \end{pmatrix} \quad (5)$$

where

$$\begin{pmatrix} I \\ I^* \\ V \\ V^* \end{pmatrix} = [I_1 I_2 \cdots I_L I_1^* I_2^* \cdots I_M V_1 V_2 \cdots V_L V_1^* V_2^* \cdots V_M^*]^T.$$

All elements in matrix of (5) are submatrices, whose orders correspond to the numbers of modes selected. If we choose L modes of electric waves and M modes of magnetic waves, then these submatrices may be expressed as follows:

$$\tilde{Z}_{[L][M]} = \begin{pmatrix} Z_{[1][1]} & Z_{[1][2]} & \cdots & Z_{[1][M]} \\ Z_{[2][1]} & Z_{[2][2]} & \cdots & Z_{[2][M]} \\ \vdots & \vdots & & \vdots \\ Z_{[L][1]} & Z_{[L][2]} & \cdots & Z_{[L][M]} \end{pmatrix}.$$

$\tilde{T}'_{(0)[\cdot]}$, $\tilde{Y}_{(0)[\cdot]}$ also take the same form and $0_{L \times M}$ is a zero matrix of order $L \times M$. The subscripts $[L]$ and (M) denote the order of the electric and magnetic waves, respectively.

In the system of equations given in (5), the propagation constant β and $[I], [I^*], [V], [V^*]$ are all unknown, but the elements of the coefficient matrix may be expressed as the integrals across the cross section of the waveguide and may be easily calculated. We use the method of calculating the eigenvalues and eigenfunctions of a matrix to solve the system of linear equations (5), and the propagation constant β and the components of the electromagnetic fields may be readily obtained.

In calculation, we use the simplified model of Fig. 2 (in which the arrows indicate the direction of magnetization approximately) instead of the actual model of Fig. 1 and assume that in all regions the magnetization is constant. This assumption is made only for simplifying calculation and in principle the coupled wave theory may be used for any complicated configurations of magnetization. From the symmetrical property of Fig. 2 and the properties of integrals of trigonometric functions it is easy to show that there are four kinds of waves which may propagate in this model:

- 1) Those whose fields are symmetrical about the y' axis and antisymmetrical about the x' axis. They may be expressed as the summation of the following modes of empty waveguide:

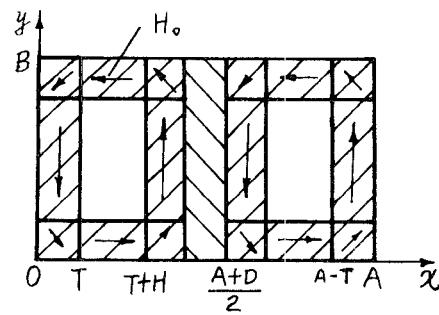


Fig. 2. Computation model of twin toroid phase shifter.

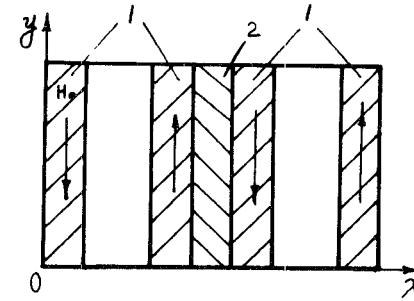


Fig. 3. Four-slab model for computation (1—ferrite slab, 2—dielectric slab).

$$H_{10}, H_{30}, H_{50}, \dots, H_{12}, H_{14}, \dots, H_{32}, H_{34} \dots \text{ and } E_{12}, E_{14}, \dots, E_{32}, E_{34} \dots$$

or the summation of H_{mn} and E_{pq} waves with m, p odd and n, q even.

- 2) Those whose fields are antisymmetrical about both the x' and y' axes and all subscripts m, n, p, q are even.
- 3) Those whose fields are symmetrical about both the x' and y' axes and all subscripts are odd.
- 4) Those whose fields are antisymmetrical about the y' axis and symmetrical about the x' axis and with subscripts m, p even and n, q odd.

It is easy to show that the dominant mode belongs to the first class of waves.

For checking the validity of the coupled wave method, we first calculate the simplified four-slab model (see Fig. 3) from the characteristic equation (exact solution) and then compare this solution with that of the coupled wave method. The characteristic equation of the four-slab model can be expressed as follows:

$$K_0 \tan(K_1 T) [M - NK_0 \tan(K_0 H)] - [Q \tan(K_1 T) - P] [\tan(K_0 H) \cdot M + K_0 N] = 0 \quad (6)$$

where

$$M = -(P^2 + Q^2) \tan(K_1 T) + [Q \tan(K_1 T) - P] K_2 \tan(K_2 L/2)$$

$$N = -\tan(K_1 T) \cdot K_2 \cdot \tan(K_2 L/2) + Q \tan(K_1 T) + P$$

$$K_2^2 = \omega^2 \epsilon_0 \mu_0 \epsilon_2 - \beta^2 \quad P = K_1 / \mu_{\perp} \\ K_0^2 = \omega^2 \epsilon_0 \mu_0 - \beta^2 \quad Q = \beta k / (\mu_{\perp} \mu) \\ K_1^2 = \omega^2 \epsilon_0 \epsilon_1 \mu_0 \mu_{\perp} - \beta^2 \quad \mu_{\perp} = (\mu^2 - k^2) / \mu.$$

TABLE I
COMPARISON OF RESULTS OF COUPLED WAVE METHOD WITH
EXACT SOLUTION FOR FOUR-SLAB MODEL (FIG. 3)

Number of modes	β_-	β_+	$\Delta\beta$
3	29.931	23.588	6.343
4	29.959	23.581	6.378
5	29.944	23.733	6.217
6	29.951	23.855	6.096
7	29.947	23.860	6.087
8	29.972	23.862	6.110
9	30.006	23.867	6.139
Exact solution	30.045	23.875	6.170

$A = 0.25, T = 0.035, D = 0.04, k_0 = 0.4, \epsilon_1 = 16, \epsilon_2 = 38$.

TABLE II
CONVERGENCE OF THE COUPLED WAVE METHOD
(COMPUTATION OF THE MODEL IN FIG. 2)

Number of modes calculated	β_-	β_+	$\Delta\beta$	Total time of computation (minutes)
40	30.098	24.978	5.120	80
20	30.055	24.967	5.088	8
16	30.057	24.971	5.086	4
14	30.062	24.981	5.081	2
11	30.074	25.017	5.057	1.8

$A = 0.25, B = 0.16, D = 0.04, T = 0.04, k_0 = 0.4, \epsilon_2 = 38, \epsilon_1 = 16$.

Here ϵ_1 is the dielectric constant of ferrite and ϵ_2 is that of dielectric slab.

III. CALCULATED RESULTS

At first we normalized all parameters used: all practical dimensions are multiplied by $1/\lambda_0$, the frequency by $1/f_0$, and the propagation constant by λ_0 , where λ_0 is the wavelength at the center frequency f_0 in free space. Besides, the antidiagonal element of the tensor permeability k is proportional to $1/f$ for the latching mode and we only list its value at the center frequency k_0 .

We first calculate the propagation constant of our phase shifter from the simplified four-slab model and then compare this solution with that of the coupled wave method. A comparison of the results of the two methods is shown in Table I. Here the order of selection of modes is $H_{10}, H_{30}, H_{50}, H_{70}, \dots$. It is clear that it is sufficient to select only the first 5–6 modes for calculation by the coupled wave method to get the necessary accuracy for practical uses.

For the practical model (Fig. 2), the convergence of the coupled wave method is shown in Table II.

The above computation is carried out on an IBM PC and it is enough to compute only 14 modes to get the necessary accuracy and saving of time.

The calculated results of the twin toroid model (Fig. 2) are shown in Figs. 4–6. The influence of the toroid wall thickness T on the phase shift versus frequency characteristic is shown in Fig. 4. ($F = f/f_0$). It is clear that when $T = 0.04$ a very flat frequency characteristic is obtained. Fig. 5 shows that the thickness of the dielectric slab also has a great influence on the phase shift versus frequency characteristic and its optimum value is also $D = 0.04$.

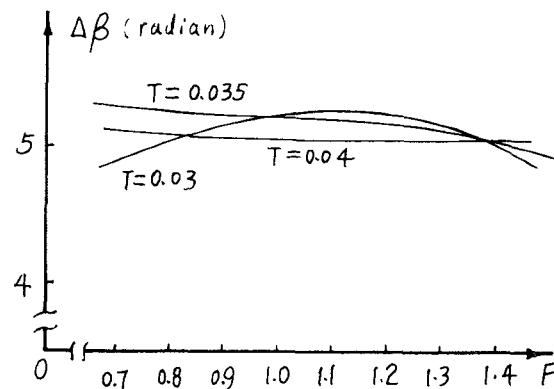


Fig. 4. Calculated phase shift versus frequency characteristic of the twin toroid model for different values of wall thickness of the toroids ($A = 0.25, B = 0.18, D = 0.04, \epsilon_1 = 16, \epsilon_2 = 38, k_0 = 0.4$)

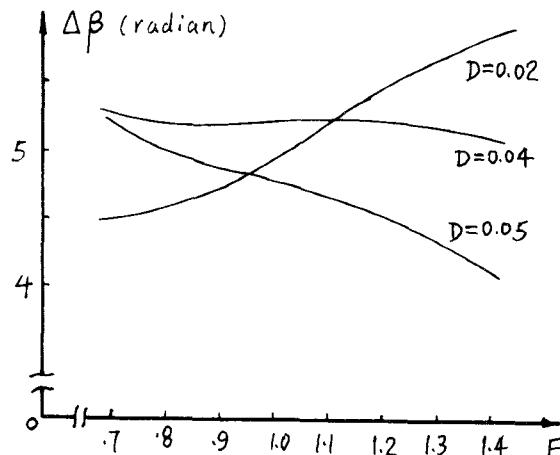


Fig. 5. Calculated phase shift versus frequency characteristics of the twin toroidal model for different values of thickness of dielectric slab ($A = 0.25, B = 0.18, T = 0.035, \epsilon_1 = 16, \epsilon_2 = 38, k_0 = 0.4$).

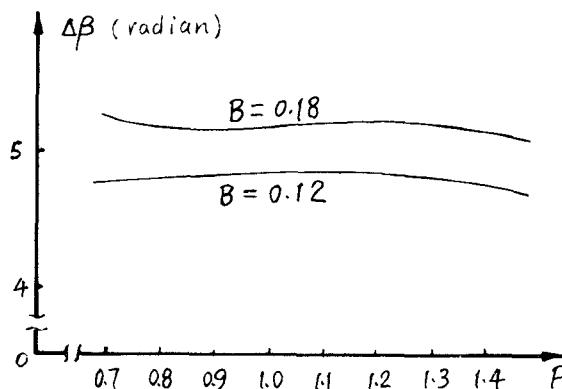


Fig. 6. Calculated phase shift versus frequency characteristics of the twin toroid model for different values of height of narrow wall of the waveguide ($A = 0.25, T = 0.035, D = 0.04, \epsilon_1 = 16, \epsilon_2 = 38, k_0 = 0.4$).

When the height of the narrow wall of the waveguide is reduced, the phase shift also decreases slightly and the phase shift versus frequency characteristic is also improved slightly, as shown in Fig. 6.

IV. EXPERIMENTAL RESULTS

Experiments on a twin toroid model are carried out and the experimental results are in good agreement with the theoretical. There is only a 3 percent discrepancy between the experiments and the calculated results using the cou-

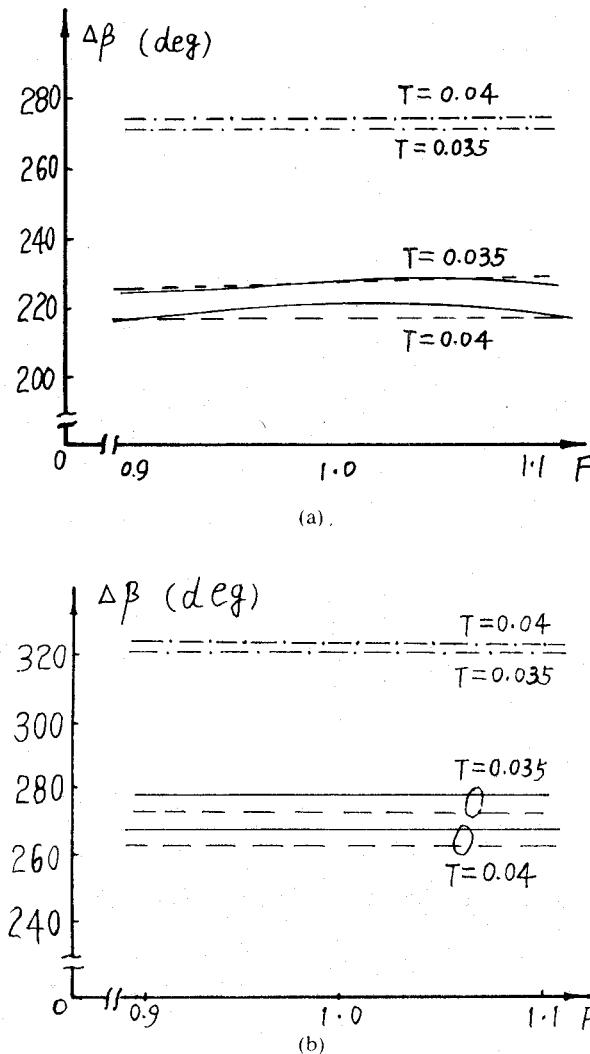


Fig. 7. Comparison of theoretical and experimental curves (a) $A = 0.25$, $B = 0.18$, $D = 0.04$, $\epsilon_1 = 16$, $\epsilon_2 = 30$, $k_0 = 0.456$. (b) $A = 0.25$, $B = 0.18$, $D = 0.04$, $\epsilon_1 = 16$, $\epsilon_2 = 38$, $k_0 = 0.456$. Length of model = $0.8\lambda_0$. — experimental curves; - - - calculated by coupled wave theory; - - - calculated by four-slab model.

pled wave method, as shown in Fig. 7. Experiments also show that the loss factor (loss in dB per 360° phase shift) of the twin toroid model is the same as or slightly less than the single toroid model when the same ferrite material and low-loss dielectric slab are used.

V. CONCLUSIONS

From the above theoretical and experimental results we can conclude the following:

- 1) The coupled wave method is very effective in calculating electromagnetic problems involving waveguides containing magnetized ferrites. In our case, it is only necessary to compute the summation of 14 modes of the empty waveguide as an approximation of the twin toroidal model. Therefore, the computation is quite simple and the computation time is rather short. It seems that this may easily be extended to other types of microwave ferrite devices, such as single toroid phase shifters and isolators.

- 2) The twin toroidal model is quite attractive in practical applications. In comparison with the single toroid model, its phase shift is considerably larger and its production is simpler. Besides, in this model the wires carrying magnetizing currents, are located in the weak microwave electric field area; hence their influence on the performance of the phase shifter is negligible and this also facilitates production. The loss factors of these two kinds of toroid phase shifters are about the same.

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